Assignment 2.

This homework is due *Thursday*, September 11.

There are total 21 points in this assignment. 16 points is considered 100%. If you go over 16 points, you will get over 100% for this homework and it will count towards your course grade (but not over 115%).

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper and *credit your collaborators*. Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers sectionss 1.2, 1.3, 2.1, 2.2 in Bartle–Sherbert.

(1) [2pt] (Paragraph 1.2.4g) Find a mistake in the following (erroneous!) induction argument:
Claim: If $n \in \mathbb{N}$ and if the maximum of the natural numbers p, q is n, then $p = q$.
"Proof." Proof by induction in n. Evidently, for $n = 1$ claim is true since in such case, $p = 1$ and $q = 1$.
Suppose, the claim holds for some $n \in \mathbb{N}$. Prove that then it also holds for $n+1$. Suppose maximum of p and q is $n+1$. Then maximum of $p-1$ and $q-1$ is $(n+1)-1 = n$. By induction hypothesis, $p-1 = q-1$, therefore $p = q$. Thus, the claim holds for $n+1$ and, by induction principle, for all natural numbers.

(2) [2pt] Prove that there does not exist a rational number r such that $r^2 = 3$.

(3) [2pt] (1.3.5) Exhibit (define explicitly) a bijection from N to the set of all odd integers greater than 13.

(4) Exhibit (define explicitly) a bijection between
- (a) [2pt] $\mathbb{Z}$ and $\mathbb{Z} \setminus \{0\}$,
- (b) [3pt, optional] $\mathbb{Q}$ and $\mathbb{Q} \setminus \{0\}$.

(5) [3pt] (1.3.12) Prove that the collection $\mathcal{F}(\mathbb{N})$ of all *finite* subsets of $\mathbb{N}$ is countable.

(6) [4pt]
- (a) ($\sim$Ex. 2.1.8a) Let x, y be rational numbers. Prove that $xy, x-y$ are rational numbers.
- (b) (Ex. 2.1.8b) Let x be a rational number, y an irrational number. Prove that $x+y$ is irrational. Prove that if, additionally, $x \neq 0$, then xy is irrational.
- (c) Let x, y be irrational numbers. Is it true that xy is always irrational? Is it true that xy is always rational?

(7) [3pt] ($\sim$Ex. 2.2.14bd,15bd) Determine and sketch the set of pairs (x, y) in $\mathbb{R} \times \mathbb{R}$ that satisfy
- (a) $|x| + |y| = 2$.
- (b) $|x| - |y| = 2$.
- (c) $|x| + |y| \geq 2$.
- (d) $|x| - |y| \geq 2$.